TRINITY COLLEGE

Year 12 Mathematics Specialist 3,4
Test 2   2021

Section 1   Calculator Free
Sketching Rational Graphs and Vectors in 3D Introduction

STUDENT'S NAME   _____

DATE: Monday 29 March   **TIME:** 19 minutes   **MARKS**: 19

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (3 marks)

Determine the angle between the two planes $r \bullet \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} = 4$ and $r \bullet \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 7$. You may express

your answer in terms of an inverse trigonometric function.
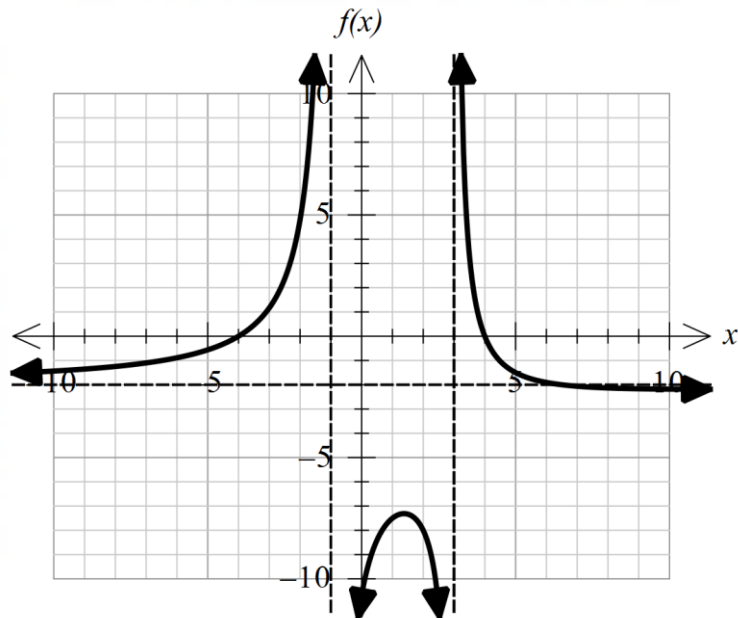
2.   (4 marks)

Determine the domain and range for $f(x) = \dfrac{1}{\sqrt{x-1}-1}$

3.   (6 marks)

The graph of $y = f(x)$ is show on the axes below. The defining rule is given by

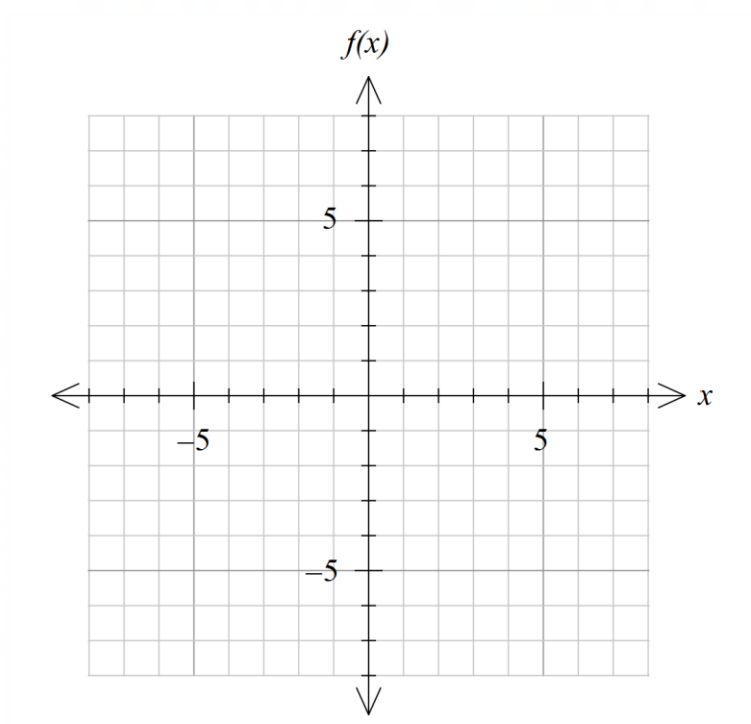$$f(x) = \frac{-a(x^2 - b)}{(x+c)(x-d)}$$ where a, b, c and d are positive constants.

Determine the value of the constants a, b, c and d. Justify your answers.

a	b	c	d

4.   (6 marks)

Sketch the function $f(x) = \dfrac{x^3 - x}{(x+2)(x-1)}$, showing all intercepts, holes, poles and asymptotes.

It is not necessary to identify any stationary points.

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Year 12 Mathematics Specialist 3,4
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Section 2   Calculator Assumed
Sketching Rational Graphs and Vectors in 3D Introduction

STUDENT'S NAME _____

DATE: Monday 29 March   **TIME:** 31 minutes   **MARKS**: 32

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this
                       assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5.   (3 marks)

(a)   Plane Π has the equation $\underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 5$ and a sphere has the vector equation $| \underset{\sim}{r} - \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} | = k$

Describe geometrically what happens for different values of k.   [2]

(b)   If the cross product of two vectors is $\underset{\sim}{0}$, describe the geometric relationship between the
two vectors.   [1]

6.   (10 marks)

A plane Π contains the three points $(1,2,3)$, $(4,5,6)$ and $(-2,3,1)$

(a)   Determine a normal to the plane Π [3]

(b)   Determine the equation of the plane Π in Cartesian form   [3]

A line, L_1, has Cartesian equation $x - 2 = y + 3 = \dfrac{z - 1}{2}$.

(c)   Determine the vector equation of the line in the form $\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$ [2]

(d)   Determine the equation of the plane that is perpendicular to plane Π and contains line L_1 [2]

7. (10 marks)

A sphere has equation $x^2 + y^2 + z^2 - 2x + 4z = 0$ and a line has equation $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$

(a) Determine the vector equation of the sphere. [3]

(b) Determine the point(s) of intersection of the line and the sphere. [4]

(c)   Determine the closest distance between the line and the centre of the sphere.   [3]

8. (9 marks)

Plane Π has Cartesian equation $y = 8x - 4z + 9$.

(a) Determine a vector normal to the plane Π. [2]

A sphere of radius 9 is tangential to the plane Π. The point $(-2, 2, 9)$ lies on the surface of the sphere. The centre of the sphere has coordinates $(-9, -2, k)$, where $k < 10$.

(a) Determine the value of k [3]

(c)   Determine the coordinates of the point of intersection of the plane Π and the sphere.

[4]